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# Bargaining to Improve Channel Sharing Between Selfish Cognitive Radios

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# Road Map

- Scenario
- Problem Formulation
- Nash Equilibrium Analysis
- Nash Bargaining Solution
- Truthfulness Consideration
- Future work: Implementation of Nash Bargaining Solution
- Conclusion

# Overview of the scenario

- We consider a case with two users sharing two channels
- Each user has his own valuation on each channel if he occupies the channel alone
- If two users share the same channel, each of them gains half of their original channel valuation
- Channel users make decisions in a distributed fashion

# Problem Formulation

- **Payoffs**

| | $C1$ | $C2$ |
|---|---|---|
| $P1$ | $a$ | $b$ |
| $P2$ | $c$ | $d$ |

TABLE I
UTILITIES WITHOUT CONFLICT

- **Original Table game**

| $P2 \setminus P1$ | $C1$ | $C2$ |
|---|---|---|
| $C1$ | $(\frac{a}{2}, \frac{c}{2})$ | $(b, c)$ |
| $C2$ | $(a, d)$ | $(\frac{b}{2}, \frac{d}{2})$ |

TABLE II
COMPLETE PAYOFF TABLE

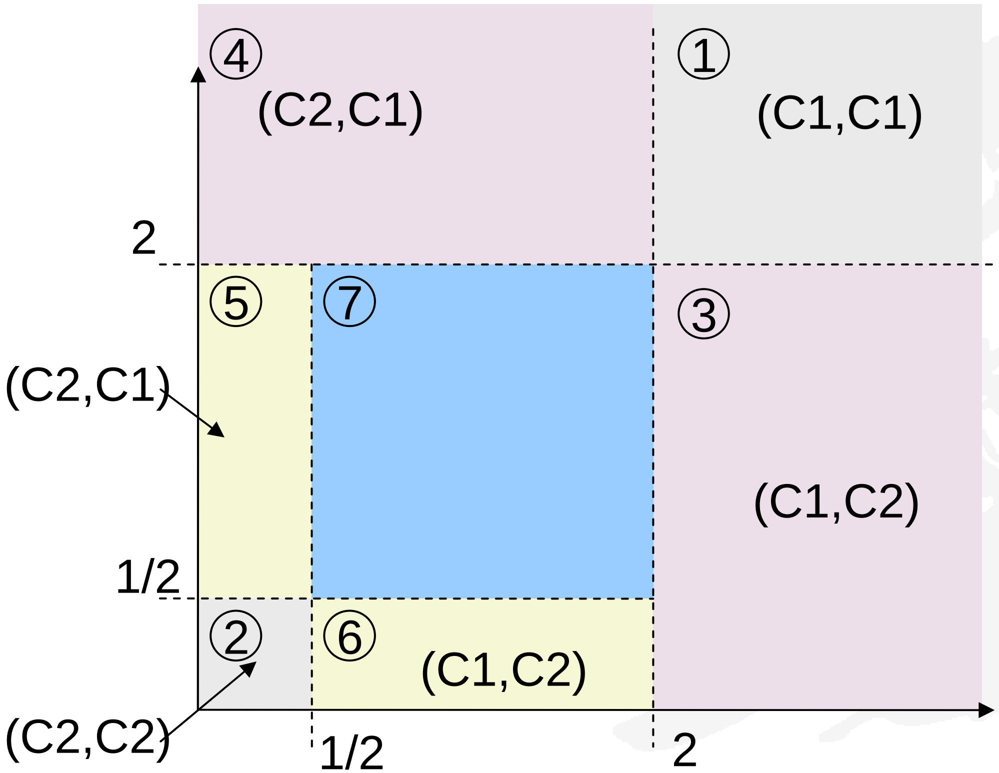
- **Affine transformations of payoffs**

| $P2 \setminus P1$ | $C1$ | $C2$ |
|---|---|---|
| $C1$ | $(\frac{a'}{2}, \frac{c'}{2})$ | $(1, c')$ |
| $C2$ | $(a', 1)$ | $(\frac{1}{2}, \frac{1}{2})$ |

TABLE III
NORMALIZED PAYOFF TABLE

# The Nash Equilibrium Analysis

- The 2D plane is divided into 7 regions



# Nash Bargaining Solution: Incentives

- Nash Bargaining solution is the only outcome that can satisfy:
  - Pareto efficiency
  - Symmetry
  - Invariance to equivalent payoff representations
  - Independence of irrelevant alternatives

# Nash Bargaining Solution: Basis

- **Convexify the payoff region: coordination signal**
  - **Time is slotted. At the beginning of each slot, the coordinator uniformly generates a random number s between 0 and 1, which is observed by both players**
  - **For the pre-agreed value α (between 0 and 1)**
    - **If s ⩽α, C1→ P1 and C2→ P2**
    - **Otherwise, C1→P2 and C2→ P1**
- **Disagreement point is the Nash Equilibrium Point**

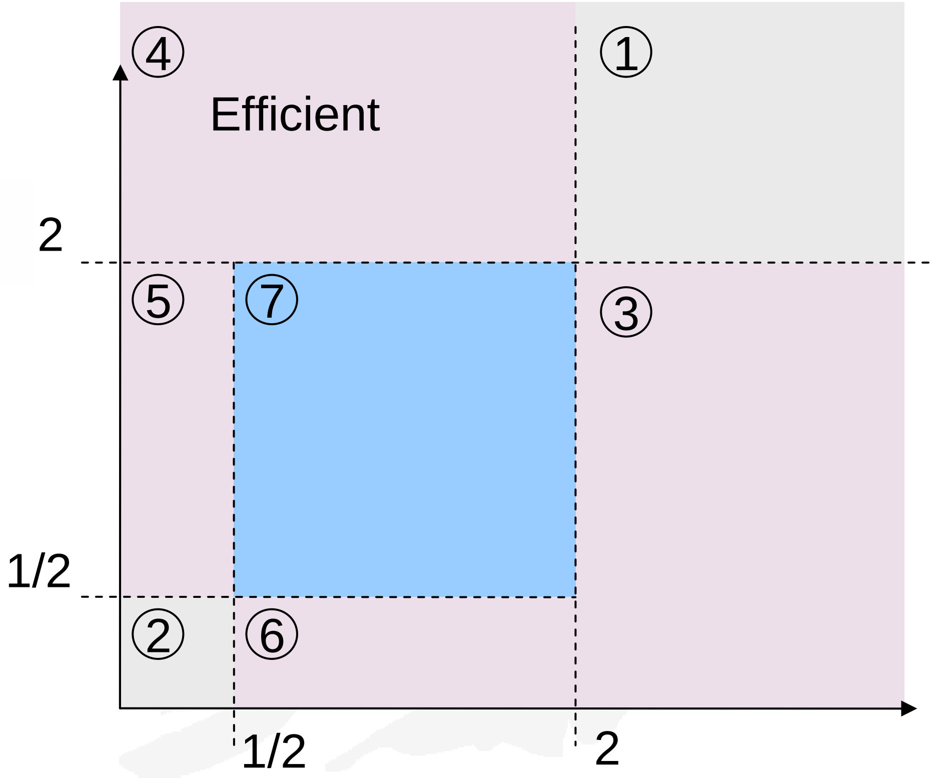
# Nash Bargaining Solution Analysis

## Formulation:

$$\max_{\alpha \in [0,1]} (u_1(\alpha) - u_1(ne))(u_2(\alpha) - u_2(ne))$$

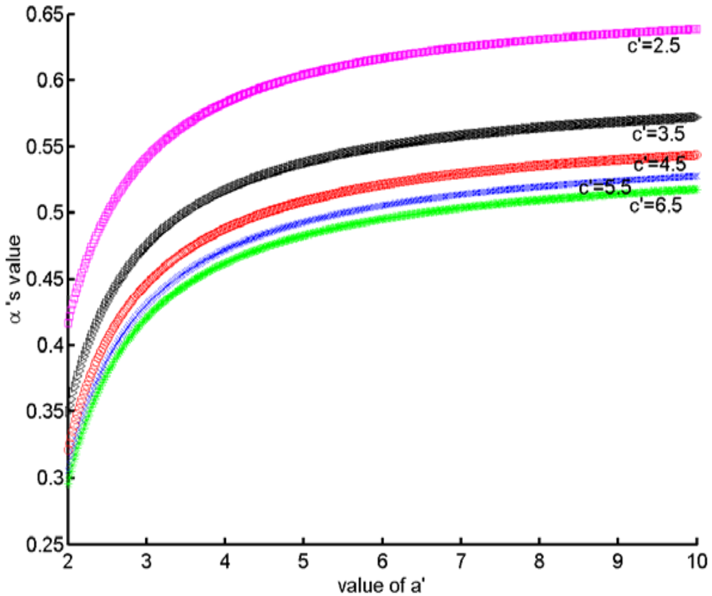
## Objective:

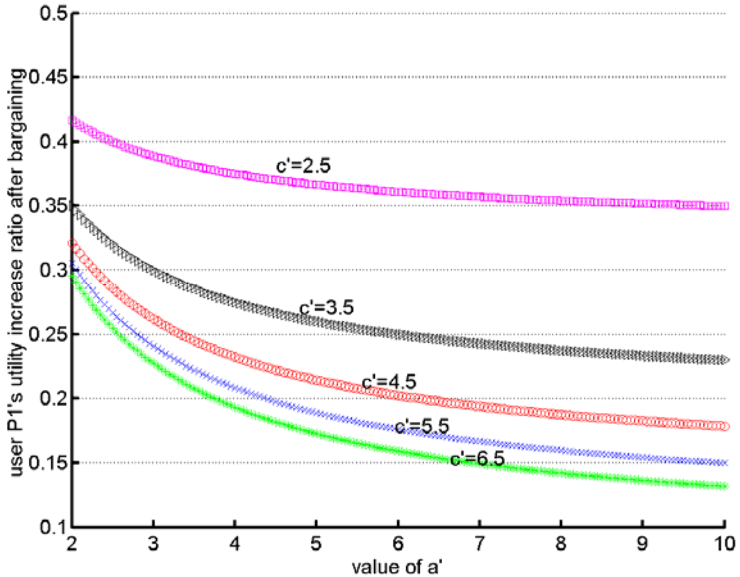
choose α such that the Nash bargaining result is maximized



# Nash Bargaining Solution Performance

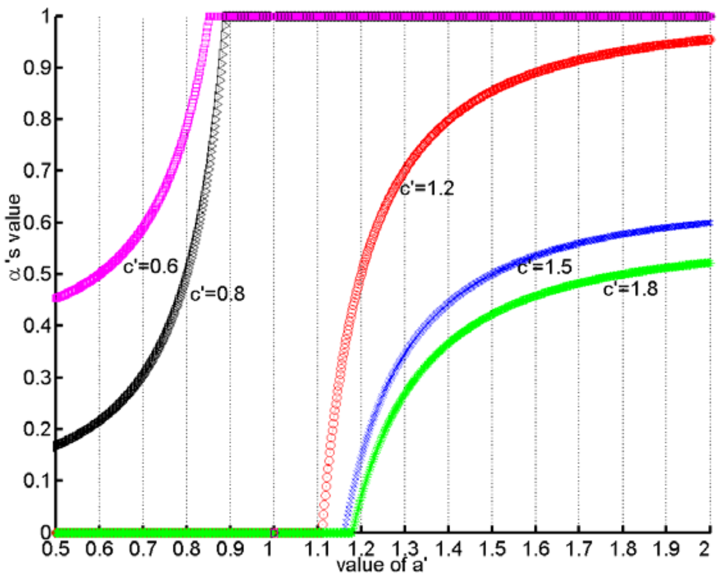
**For case 1:**

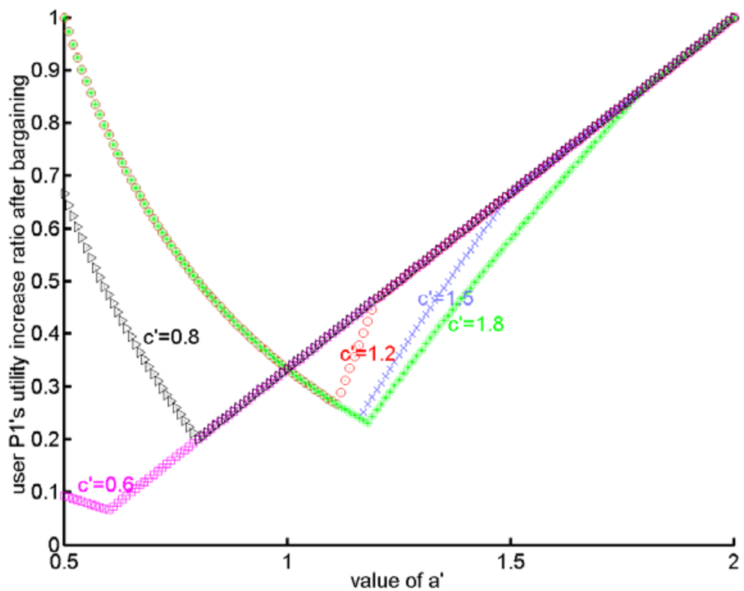


# Nash Bargaining Solution Performance

- **For case 7**



# Truthfulness Consideration

- Motivation to consider truthfulness
- Model the user's "behavior" and "belief"
  - Behavior (objectively):
    - Lying
    - Truth-telling
  - Beliefs (subjectively):
    - Suspicious
    - Gullible

# Three truthfulness Models

- Three truthfulness models:
  - M1: Lying prone model: if a user will not lose anything by lying, he/she will lie
  - M2: Neutral model: if a user can possibly gain and never lose by lying, the user will lie
  - M3: Truth telling prone model: if a user doesn't lose by telling the truth, he/she will NOT lie

# Neutral Model Analysis

- We consider M2 (Neutral Model)
- In this particular problem, a user will lie *if and only if* the following two conditions hold:
  - Incentive Condition
  - Risk Aversion Condition
- Two theorems

# Two theorems about truthfulness with M2

- Theorem 1
  - In the non-cooperative game with the gullible user assumption, truthfulness for both users is ensured under the neutral model (M2)
- Theorem 2
  - Truthfulness is not ensured in current Nash bargaining mechanism under the neutral model (M2)

# Conclusions on truthfulness consideration

- Truthfully reporting channel valuations is not incentivized in the current Nash bargaining mechanism
- To implement the Nash bargaining solution, new mechanism is needed
- Nash implementation of the Nash bargaining solution

# Nash Implementation

- Nash implementation of the Nash bargaining solution:
  - Nash implementation is not a dominant strategy implementation. Therefore, it does not guarantee truthfulness. Instead, Nash implementation guarantees that with rational players, the outcome has to be the Nash bargaining solution.
  - An extensive game form with perfect information and chance moves can implement the Nash bargaining solution exactly

# Nash Implementation for Two Players

- SPE implementation of the Nash bargaining Solution
  - Phase 1
    - Player 1 specifies a point X
    - Player 2 specifies a point Y
  - Phase 2: A trial between X and Y
    - Player 1 specifies a real number r between [0,1]
    - Player 2 may concede, challenge or counter by specify t $r \leqslant t \leqslant 1$
      - If player 2 concedes, X is the chosen point from this phase
      - If player 2 challenges, 1 must concede (in which case Y is chosen), or else specify r'>r and allow 2 to choose between r'X and Y
      - If player 2 conters, 1 may choose between tX and Y

Reference: J.V. Howard, "A social choice rule and its implementation in perfect equilibrium", JET '92, 142-159

# Nash Implementation for Two Players

- Phase 3
  - Player 1 may alter the chosen point to Q
  - Player 2 may alter the chosen point to Q
- Phase 4
  - We call it "necessary" only if r'Y or tX or Q has been chosen. If it is necessary, the players in turn specify a point. If player i specifies q_i, then Q is ½ (q_1 + q_2)

Reference: implementation for three and more players can be found in Naeve, Jörg "Nash Implementation of the Nash Bargaining Solution by a Natural Mechanism ", 1998

# Conclusions and Future Work

- **Conclusions**
  - Formulated the channel sharing game
  - Analyzed the Nash equilibrium of the game
  - For the inefficient Nash equilibria, propose Nash Bargaining solution with a coordination signal. NBS guarantees 100% utilization of the channel resource.
  - Discussed truthfulness of the Nash bargaining solution

- **Future work:**
  - Nash implementation of the Nash bargaining solution
  - Multiple dimension cases: multiple users and multiple channels